

**A HIERARCHY OF EFFECTIVE FIELD THEORIES
OF HOT ELECTROWEAK MATTER**A. Jakovác^a, K. Kajantie^b and A. Patkós^{a,c}^a *Dept. of Atomic Physics, Eötvös Univ., Budapest*^b *Dept. of Theor. Phys., University of Helsinki*^c *The Niels Bohr Institute, University of Copenhagen*

Abstract: A hierarchy of effective three-dimensional theories of finite temperature electroweak matter is studied. First an integration over non-static modes leads to an effective theory containing a gauge field A_i^a , an adjoint Higgs field A_0^a and the fundamental Higgs field ϕ_α . We carry out the integration in the 1-loop approximation, study renormalisation effects and estimate quantitatively those terms of the effective action which are suppressed by inverse powers of the temperature. Secondly, because of the existence of well-separated thermal mass scales, A_0^a can be integrated over, and finally also ϕ_α , leaving an effective theory of the A_i^a . In the analysis of the subsequent models particular attention is paid to the screening of the magnetic fluctuations due to the integrated-out degrees of freedom.

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1. Introduction

The restoration of the global gauge symmetry of electroweak interactions appears to be a particularly weak first order phenomenon[1-5]. The general experience with phase transition phenomena gives the intuition that the lightest thermal fluctuations (of the longest correlation length) play decisive role in the transition range. Renormalisation group techniques suggest that the role of the more "massive" fluctuations is to influence the effective dynamics of the light modes. In the standard model of electroweak interactions one encounters several mass scales, all varying with temperature. Therefore the search for the existence of scale hierarchies should be realised in the process of solving the model.

From this point of view the elimination of the non-zero Matsubara frequency modes [6,7] represents only a first step of the gradual reduction. It rearranges the usual perturbative series of the effective potential. For instance, even if the integration is performed in the Gaussian approximation, it brings into the tree-level expression of the effective three-dimensional potential perturbatively higher order contributions. In section 2 of the present paper we are going to discuss in great detail the importance of different terms of the effective Lagrangian, arising from the expansion of the 1-loop expression of the contribution from the non-static modes.

The main tool of this investigation is the comparison of the characteristics of the electroweak transition computed perturbatively within different versions of the effective 3D theory. The leading result of two subsequent 1-loop integrations is known to be equivalent to the summation of the leading infrared sensitive loop contributions in the 4D thermal theory. Subleading effects related to taking into account higher dimensional field combinations in the effective 3D theory and/or to the implementation of specific 4D renormalisation conditions for the T -independent part of the potential energy density, are discussed for different ranges of the Higgs-mass. Efforts will be made to clarify relevant technical details of the evaluation and renormalisation of the 1-loop fluctuation determinant.

The general conclusion of this part of our study is that the subleading corrections have increasingly important impact on the transition as the Higgs mass is lowered. However, for $m_H > 50$ GeV one can safely omit them from the effective model.

The perturbative solution of the 3D model suggests that distinct mass-scales exist also among the static fluctuation modes. In section 3 further thinning of the degrees of freedom will be proposed on the basis of this observation. The first field to be eliminated is the adjoint Higgs field A_0^a . It gets screened already through the integration of the non-static modes, and is found to be much more massive than other fluctuations for any Higgs mass.

In the experimentally relevant range $60 \text{ GeV} \leq m_H$ the existence of further distinct scales is not obvious. Only above 80 GeV can one convincingly argue that the perturbatively determined thermal masses of the scalar ϕ fields are 3-4 times larger near T_c than $m_W(T = T_c)$. Then the integration over these scalar fields is also justified. The main results of this part of our paper are the propositions for effective 3D gauge field-Higgs and pure gauge theories (Eqs. (29) and (34)), respectively.

In the course of the gradual integrations one can observe many pieces of the infrared improved self-energies, familiar from the corresponding 1-loop Schwinger-Dyson equations of the full thermal theory [3,4], to emerge. Moreover, the non-zero scalar background breaking the global gauge symmetry, induces not just a piece of mass for the magnetic vector potential but also higher " A^{2n} " potential terms do appear. It is remarkable that working with non-Abelian constant vector background the finite T rescaling of the gauge kinetic term ("wave-function renormalisation") can be obtained, too.

Non-perturbative methods are being applied to the study of the 3D effective model of the electroweak transition [8,9], with indications for important non-perturbative effects. The effective 3D gauge field-Higgs and pure gauge field models resulting from successively integrating out A_0^a and ϕ would merit non-perturbative studies of their own.

Section 4 is devoted to the discussion of the description of the electroweak transition emerging from the perturbative solution of the effective pure gauge theory. In spite of the crudeness of the approximation it predicts stronger discontinuities at the phase transition in comparison with the perturbative solution of the full finite T theory. This feature might hint to the true characteristics of the more complete description.

On the basis of the present work the region of small (≤ 30 GeV) Higgs masses cannot be described adequately with a pure gauge effective model. We found evidence that the appropriate theory of the system in this regime is rather an effective scalar model. The investigation of this theory with help of non-perturbative numerical methods is the subject of a parallel project [11].

2. Virtues of the effective theory of static modes

The decoupling of non-zero frequency (non-static) Matsubara modes from the high temperature thermodynamics of field theories has been proposed long time ago [6] and has been studied in detail, for example, in [7].

It is well-known, that the leading thermal (screening) masses are correctly induced by integrating over the non-static modes at 1-loop. It is, however, less widely appreciated that the 1-loop Schwinger-Dyson equations for the self-energies of the different fields, as calculated from the effective 3D theory coincide with the 1-loop equations of the full thermal theory. This means, that the infrared improvement program for the perturbation theory of the effective Higgs-potential [3,4] can be implemented in the effective model, with equivalent results.

From our point of view the role of the integration over the non-static modes is not only to improve the infrared behavior of the propagators, but also to induce new, highly non-linear interactions between the static fields. These arise when the logarithm of the determinant of the 1-loop functional fluctuations is expanded into power series with respect to the background fields. At $T = \infty$ all combinations with dimensions higher than 4 are negligible. The appearance of these terms (formally of order g^6, λ^3) in the tree-level 3D action illustrates how the gradual integration reorganizes the perturbative series. At the realistic electroweak phase transition the importance of these terms should be quantitatively assessed. An attempt for this in the U(1) and SU(2) Higgs models is the content of this section.

The models under consideration are the following:

G=SU(2):

$$S = \int_0^\beta \int d^3x \left[\frac{1}{4} F_{mn}^a F_{mn}^a + \frac{1}{2} (D_m \Phi)^+ (D_m \Phi) + \frac{1}{2} m^2 \Phi^+ \Phi + \frac{\lambda}{24} (\Phi^+ \Phi)^2 \right] + \text{counter terms}, \quad (1)$$

G=U(1):

$$S = \int_0^\beta \int d^3x \left[\frac{1}{4} F_{mn} F_{mn} + \frac{1}{2} |(\partial_m + ig A_m) \Phi|^2 + \frac{1}{2} m^2 |\Phi|^2 + \frac{\lambda}{24} |\Phi|^4 \right] + \text{counter terms} \quad (2)$$

($m=1, \dots, 4$; $a=1, 2, 3$; $D_m \Phi = (\partial_m + ig A_m^a \tau^a / 2) \Phi$).

For definiteness the integration over the non-static fields will be discussed in considerable detail for the G=SU(2) case, the analogous formulae for G=U(1) will be just listed next to it.

The elimination of the non-static modes is most conveniently performed in the thermal static gauge, which preserves invariance under space-dependent gauge transformations:

$$A_0^a(x, \tau) = A_0^a(x) + a_0^a(x, \tau), \quad a_0^a(x, \tau) = \sum_{n \neq 0} a_{0,n}^a(x) e^{i\omega_n \tau}, \quad \omega_n = 2\pi T n,$$

$$a_0^a(x, \tau) \equiv 0, \quad A_0^a(x) \equiv \delta^{a3} A_0(x). \quad (3)$$

Then, it is sufficient to keep in the computations of the effective action only the static background fields $A_0(x)$, $\Phi_0(x)$, since the dependence on static $A_i(x)$ is easily established with help of the minimal coupling principle. Furthermore, we are going to investigate only the potential part of the induced action, therefore in the course of the actual calculations the background fields will be treated as constants.

Performing the expansion of (1) in the non-static fields up to quadratic terms, the fluctuations, coupled on the Gaussian level are separated most conveniently by introducing the spatially transverse ($a_{i,T}$) and longitudinal ($a_L \equiv \partial_i a_i$) components of the vector fluctuations. Then the groups fluctuating independently in the gauge (3) are $(a_L^1, a_L^2, \xi_1, \xi_2)$; (a_L^3, η_1, η_2) ; $(a_{i,T}^1, a_{i,T}^2)$; $a_{i,T}^3$, where the fluctuations of the Higgs field around the background are parametrized as

$$\Phi = (\xi_1 + i\xi_2, \Phi_0 + \eta_1 + i\eta_2).$$

The matrices of the quadratic forms for the fluctuations are the following, respectively:

$$\begin{aligned} K_1 &= \begin{pmatrix} m_L^2 & 2igA_0\omega_n & 0 & \frac{ig}{2}\Phi_0 k \\ -2ieA_0\omega_n & m_L^2 & \frac{ig}{2}\Phi_0 k & 0 \\ 0 & -\frac{ig}{2}\Phi_0 k & M^2 & -igA_0\omega_n \\ -\frac{ig}{2}\Phi_0 k & 0 & igA_0\omega_n & M^2 \end{pmatrix}, \\ K_2 &= \begin{pmatrix} m_L'^2 & 0 & -\frac{ig}{2}\Phi_0 k \\ 0 & M'^2 & igA_0\omega_n \\ \frac{ig}{2}\Phi_0 k & -igA_0\omega_n & M^2 \end{pmatrix}, \\ K_3 &= \begin{pmatrix} m_T^2 & 2igA_0\omega_n \\ -2igA_0\omega_n & m_T^2 \end{pmatrix}, \\ K_4 &= (m_T'^2), \end{aligned} \quad (6)$$

where we use the temporary abbreviations

$$\begin{aligned} m_L^2 &= \omega_n^2 + g^2 \left(A_0^2 + \frac{\Phi_0^2}{4} \right), \\ m_L'^2 &= \omega_n^2 + g^2 \frac{\Phi_0^2}{4}, \\ m_T^2 &= k^2 + \omega_n^2 + g^2 \left(A_0^2 + \frac{\Phi_0^2}{4} \right), \\ m_T'^2 &= k^2 + \omega_n^2 + g^2 \frac{\Phi_0^2}{4}, \\ M^2 &= k^2 + \omega_n^2 + m^2 + g^2 \frac{A_0^2}{4} + \lambda \frac{\Phi_0^2}{6}, \\ M'^2 &= k^2 + \omega_n^2 + m^2 + g^2 \frac{A_0^2}{4} + \lambda \frac{\Phi_0^2}{2}. \end{aligned} \quad (7)$$

After evaluating the four functional determinants, the expression of the 1-loop non-static contribution to the non-derivative part of the 3D action can be given as follows:

$$\begin{aligned} U(A_0, \Phi) &= \sum_{n \neq 0} \sum_k \left\{ \ln(K^2 + \frac{1}{4}g^2\Phi_0^2) + \ln[(K^2 + g^2(A_0^2 + \frac{1}{4}\Phi_0^2))^2 - 4g^2A_0^2\omega_n^2] \right. \\ &+ \frac{1}{2} \ln[(\omega_n^2 + \frac{1}{4}g^2\Phi_0^2)((K^2 + \frac{1}{4}g^2A_0^2 + \frac{1}{6}\lambda\Phi_0^2 + m^2)(K^2 + \frac{1}{4}g^2A_0^2 + \frac{1}{2}\lambda\Phi_0^2 + m^2) \\ &\quad \left. - g^2A_0^2\omega_n^2) - \frac{1}{4}g^2\Phi_0^2k^2(K^2 + \frac{1}{4}g^2A_0^2 + \frac{1}{2}\lambda\Phi_0^2 + m^2)] \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \ln [(\omega_n^2 + g^2(A_0^2 + \frac{1}{4}\Phi_0^2))^2 ((K^2 + \frac{1}{4}g^2A_0^2 + \frac{1}{6}\lambda\Phi_0^2 + m^2)^2 - g^2\omega_n^2A_0^2) \\
& - \frac{1}{2}g^2\Phi_0^2k^2(\omega_n^2 + g^2(A_0^2 + \frac{1}{4}\Phi_0^2))(K^2 + \frac{1}{4}g^2A_0^2 + \frac{1}{6}\lambda\Phi_0^2 + m^2) + 4g^4A_0^4\omega_n^4 \\
& - 4g^2A_0^2\omega_n^2(K^2 + \frac{1}{4}g^2A_0^2 + \frac{1}{6}\lambda\Phi_0^2 + m^2)^2 - g^4A_0^2\Phi_0^2\omega_n^2k^2 + \frac{1}{16}g^4\Phi_0^2k^4] \}. \quad (8)
\end{aligned}$$

In this equation $K^2 = k^2 + \omega_n^2$.

In order to find the couplings for the various pieces of the effective theory one expands the logarithms in polynomials of the background fields. The expansion is fully justified, since the $n = 0$ modes are left out from the calculation of the sums. This rather cumbersome calculation is outlined for the more transparent case $A_0 = 0$ in the Appendix. This case is actually sufficient to treat the renormalisation of the action. For this we use the analogue of Linde's renormalisation conditions [10,12]:

$$\frac{dU(T - \text{indep})}{d\Phi_0} = 0, \quad \frac{d^2U(T - \text{indep})}{d^2\Phi_0} = m_H^2(T = 0), \quad \phi = v_0. \quad (9)$$

Here v_0 is the classical vacuum expectation value of the Higgs field and $m_H(T = 0)$ is the physical Higgs mass.

The cut-off regularised expression of the scalar potential is given in eq.(A.7). For renormalisation its T -dependent part should be separated. This is achieved by rewriting the term depending logarithmically on Λ^2/T^2 in the following convenient form:

$$\begin{aligned}
& - \frac{1}{64\pi^2} \ln \frac{\Lambda^2}{T^2} [m^2\Phi_0^2(\frac{9g^2}{2} + 2\lambda) + \Phi_0^4(\frac{9g^4}{16} + \frac{3g^2\lambda}{4} + \frac{\lambda^2}{3})] \\
& = - \frac{1}{64\pi^2} \sum_q n_q m_q^4(\Phi_0) [\ln \frac{\Lambda^2}{m_q^2(v_0)} + \ln \frac{m_q^2(v_0)}{m_q^2(\Phi_0)} + \ln \frac{m_q^2(\Phi_0)}{T^2}]. \quad (10)
\end{aligned}$$

Here q runs through the fields of the theory: $q = T$ (transversal), L (longitudinal), H (Higgs), G (Goldstone), the $m_q^2(\Phi_0)$ are appropriately chosen mass-squared expressions given below and the n_q are the corresponding multiplicities.

Choosing for the physical fields (the transversal gauge and the Higgs) the familiar mass expressions (coinciding with the $T = 0$ formulae when $\Phi_0 = v_0$ is substituted)

$$\begin{aligned}
n_T &= 6, \quad m_T^2(\Phi_0) = \frac{1}{4}g^2\Phi_0^2, \\
n_H &= 1, \quad m_H^2(\Phi_0) = m^2 + \frac{\lambda}{2}\Phi_0^2, \quad (11)
\end{aligned}$$

then the transcription (10) requires the unique further choice

$$\begin{aligned}
n_L &= -3, \quad m_L^2(\Phi_0) = \frac{1}{\sqrt{2}}g^2\Phi_0^2, \\
n_G &= 3, \quad m_G^2(\Phi_0) = m^2 + (\frac{3}{4}g^2 + \frac{\lambda}{6})\Phi_0^2. \quad (12)
\end{aligned}$$

(Recall from the Appendix, that the fluctuations in these last degrees of freedom are not eigendirections of the fluctuation matrix, therefore these "masses" have no direct physical meaning, not even for $T = 0$.)

The last term in the square bracket of the right hand side of (10) ($\sim \ln(m_q^2(\Phi_0)/T^2)$) will be associated with the temperature dependent part of the effective potential. The first term clearly is to be absorbed into the counterterms, while the second gives a finite contribution. Since the

renormalisation conditions (9) are fulfilled by the non-logarithmic terms separately, also the first two terms of the expansion of this finite term around $\Phi_0 = v_0$ should be cancelled by the counterterms. Therefore the T -independent part of the potential fulfilling the renormalisation conditions reads as

$$U_{\text{eff}}(T - \text{indep}) = \frac{1}{2}m^2\Phi^+\Phi + \frac{1}{4}(\Phi^+\Phi)^2 + \frac{1}{64\pi^2} \sum_q n_q [m_q^4(\Phi)(\ln \frac{m_q^2(\Phi)}{m_q^2(v_0)} - \frac{3}{2}) + 2m_q^2(\Phi)m_q^2(v_0)]. \quad (13)$$

Upon adding to it the T -dependent part from eq. (10), the term $\ln m_q^2(\Phi)/m_q^2(v_0)$ of (13) will be replaced by $\ln T^2/m_q^2(v_0)$.

The final expression for the effective action with all abbreviations written out explicitly is given in the following eqs.(14-17):

$$S[\bar{A}_i, \bar{A}_0, \varphi] = \int d^3x [L_{\text{kin}} + U(\bar{A}_0, \varphi)],$$

where

$$L_{\text{kin}} = \frac{1}{4}\bar{F}_{ij}^a\bar{F}_{ij}^a + \frac{1}{2}[(\partial_i + i\bar{g}\bar{A}_i)\varphi]^+[(\partial + i\bar{g}\bar{A}_i)\varphi] + \frac{1}{2}(\partial_i\bar{A}_0 + \bar{g}\epsilon^{abc}\bar{A}_i^b\bar{A}_0^c)^2, \\ U_{\text{dim4}}(\bar{A}_0, \varphi) = \frac{1}{2}m_\varphi^2\varphi^+\varphi + \frac{1}{2}m_D^2\bar{A}_0^2 + \frac{1}{8}\bar{g}^2\bar{A}_0^2\varphi^+\varphi + \frac{T\hat{\lambda}}{24}(\varphi^+\varphi)^2 + \frac{17\beta\bar{g}^4}{192\pi^2}(\bar{A}_0^2)^2 + \text{3D counterterms}, \quad (14)$$

$$m_\varphi^2 = \hat{m}^2 + (\frac{3}{16}\bar{g}^2 + \frac{\bar{\lambda}}{12})T, \quad m_D^2 = \frac{5}{6}\bar{g}^2T + \frac{m_R^2\bar{g}^2}{8\pi^2T}, \quad (15)$$

$$\hat{m}^2 = m^2(1 - \frac{1}{32\pi^2}((\frac{9}{2}g^2 + \lambda)\ln \frac{3g^2v_0^2}{4T^2} + \lambda\ln \frac{\lambda v_0^2}{3T^2}) - \frac{1}{128\pi^2}(45g^2 + 20\lambda + \frac{27g^4}{\lambda})), \\ \hat{\lambda} = \lambda - \frac{3}{8\pi^2}(g^4\frac{3}{8}(\ln \frac{g^2v_0^2}{4T^2} - \frac{3}{2}\ln \frac{g^2v_0^2}{\sqrt{2}T^2}) + \frac{\lambda^2}{4}\ln \frac{\lambda v_0^2}{3T^2} + 3(\frac{3g^2}{4} + \frac{\lambda}{6})^2\ln \frac{3g^2v_0^2}{4T^2}) - \frac{9}{16\pi^2}(\frac{9g^4}{16} + \frac{3g^2\lambda}{4} + \frac{\lambda^2}{3}). \quad (16)$$

In eqs.(14-16) we have introduced rescaled 3D fields and couplings, with dimensionalities appropriate to a 3D theory:

$$\varphi = \sqrt{\beta}\Phi, \quad \bar{A}_i = \sqrt{\beta}A_i, \quad \bar{A}_0 = \sqrt{\beta}A_0, \quad \bar{g}^2 = g^2T, \quad \bar{\lambda} = \lambda T. \quad (17)$$

The 3D counterterms, fully specified by the reduction step, will cancel linearly diverging radiative mass contributions to the φ and \bar{A}_0 fields of the effective 3D theory. The form (14) of the effective theory with only the leading $\mathcal{O}(T^2)$ mass corrections and unmodified λ has been given in [8].

The magnitude of the logarithmic corrections to m^2 and λ is illustrated by Fig. 1, where in the $m_H - T$ -plane the contour lines corresponding to 3%, 5% and 7% variations are displayed. The critical region for the interesting range of Higgs masses is covered by these lines. The effect of renormalisation conditions on the physical characteristics of the transition will be discussed further at the end of this section.

The corrections of relative order m^2/T^2 and $g^2\Phi^2/T^2$ can be calculated by retaining one more order in the expansions of the logarithms of (8). The corresponding dim 6 combinations of m^2, A_0^2 and Φ^2 provide two types of contributions to the effective 3D theory. Firstly, $\mathcal{O}(m^2/T^2)$ corrections

will be generated to the thermal masses of (15) as well as to the couplings $\bar{\lambda}$ and \bar{g} . Secondly, new interactions of the general form

$$U_{\text{dim } 6}(\bar{A}_0, \varphi) = \frac{1}{T^3} [a(\varphi^+ \varphi)^3 + b(\varphi^+ \varphi)^2 \bar{A}_0^2 + c\varphi^+ \varphi (\bar{A}_0^2)^2 + d(\bar{A}_0^2)^3]. \quad (18)$$

are also produced. After performing the necessary algebraic manipulations with help of symbolic program packages one finds the following results for these new contributions:

$$\begin{aligned} \delta m_\phi^2 &= \frac{m^4 \zeta(3)}{128 T^3 \pi^4} (-\frac{9}{4} \bar{g}^2 + \bar{\lambda}), \\ \delta m_D^2 &= -\frac{m^4 \zeta(3) \bar{g}^2}{64 T^3 \pi^4}, \\ \delta \bar{\lambda} &= \frac{m^2 \zeta(3)}{32 T^3 \pi^4} (-\frac{27}{16} \bar{g}^4 - \frac{9 \bar{g}^2 \bar{\lambda}}{4} + \bar{\lambda}^2), \\ \delta \bar{g}^2 &= -\frac{m^2 \zeta(3) \bar{g}^2}{8 T^3 \pi^4} (3 \bar{g}^2 + \frac{1}{4} \bar{\lambda}), \\ a &= \frac{\zeta(3)}{1024 \pi^4} (\frac{3}{16} \bar{g}^6 - \frac{3}{8} \bar{\lambda} \bar{g}^4 - \frac{1}{4} \bar{\lambda}^2 \bar{g}^2 + \frac{5}{27} \bar{\lambda}^3), \\ b &= -\frac{\zeta(3) \bar{g}^2}{1024 \pi^4} (\frac{109}{16} \bar{g}^4 + \frac{47}{6} \bar{\lambda} \bar{g}^2 + \frac{5}{9} \bar{\lambda}^2), \\ c &= -\frac{\zeta(3) \bar{g}^6}{64 \pi^4}, \quad d = 0. \end{aligned} \quad (19)$$

(Here $\delta \bar{\lambda}$ and $\delta \bar{g}^2$ refer to the finite corrections of the coefficients in front of $(\varphi^+ \varphi)^2/24$ and $\bar{A}_0^2 \varphi^+ \varphi/8$, respectively.)

At this stage we can make three comments on the result. Firstly, note that identically $d = 0$, there is no direct induced sixth order selfcoupling for A_0 . Secondly, the numerical coefficients in front of the couplings are small, which diminishes the impact of these corrections even in the non-asymptotic coupling range. Finally, one observes that the ground state $\varphi = \varphi_0, \bar{A}_0 = 0$ might become unstable for large values of φ_0 in certain ranges of the couplings due to the the sixth power correction to the potential. This is probably compensated by higher powers of the expansion, as has been observed also in other cases [7]. It does not cause here any problem, since this piece of the potential turns out not to be important for the range of φ -values of interest for the phase transition. In the range of the couplings $\lambda \ll g^2$, where the sixth power term plays observable role, its coefficient is positive.

In case of U(1) theory an expression parallel to (8) has been published before in [13]. The result including the dim 4 part of the potential is:

$$\begin{aligned} L_{\text{kin}}^{U(1)} &= \frac{1}{4} \bar{F}_{ij} \bar{F}_{ij} + \frac{1}{2} |(\partial_i + i \bar{g} \bar{A}_i) \varphi|^2 + \frac{1}{2} (\partial_i \bar{A}_0)^2, \\ U_{\text{dim } 4}^{U(1)}(\bar{A}_0, \varphi) &= \frac{1}{2} m_\phi^2 |\varphi|^2 + \frac{1}{2} m_D^2 \bar{A}_0^2 + \frac{T \hat{\lambda}}{24} |\varphi|^4 + \frac{\bar{g}^4 \beta}{24 \pi^4} \bar{A}_0^4 + \frac{1}{2} \bar{g}^2 \bar{A}_0^2 |\varphi|^2 \\ &\quad + 3\text{D counterterms}, \end{aligned} \quad (20)$$

where the fields and the couplings are related to their 4D counterparts as in (17), while

$$\begin{aligned} m_\phi^2 &= \hat{m}^2 + (\frac{2 \bar{\lambda}}{3} + 3 \bar{g}^2) \frac{T}{12}, \quad m_D^2 = \bar{g}^2 (\frac{T}{3} + \frac{m^2}{4 \pi^2 T}), \\ \hat{m}^2 &= m^2 (1 - \frac{1}{8 \pi^2} (\frac{9 g^4}{2 \lambda} + \frac{2 \lambda}{2} + 2 g^2) + \frac{1}{16 \pi^2} (\lambda \ln \frac{\lambda v_0^2}{3 T^2} + 2 (3 g^2 + \frac{\lambda}{6}) \ln \frac{3 g^2 v_0^2}{T^2})), \end{aligned}$$

$$\begin{aligned}\hat{\lambda} &= \lambda - \frac{9}{16\pi^2}(3g^4 + g^2\lambda + \frac{5g^2}{18}) \\ &+ \frac{3}{8\pi^2}(2g^4(\ln \frac{g^2 v_0^2}{T^2} - 4\ln \frac{2\sqrt{2}g^2 v_0^2}{T^2}) + \frac{\lambda^2}{4} \ln \frac{\lambda v_0^2}{3T^2} + (3g^2 + \frac{\lambda}{6})^2 \ln \frac{3g^2 v_0^2}{T^2}).\end{aligned}\quad (21)$$

The list of dim 6 corrections in case of the Abelian Higgs model is the following:

$$\begin{aligned}\delta m_\phi^2 &= \frac{m^4 \zeta(3)}{384T^3 \pi^4}(-9\bar{g}^2 + \bar{\lambda}), \\ \delta m_D^2 &= -\frac{m^4 \zeta(3) \bar{g}^2}{32T^3 \pi^4}, \\ \delta \bar{\lambda} &= \frac{m^2 \zeta(3)}{32T^3 \pi^4}(-9\bar{g}^4 - 3\bar{g}^2 \bar{\lambda} + \frac{5}{6}\bar{\lambda}^2), \\ \delta \bar{g}^2 &= -\frac{m^2 \zeta(3) \bar{g}^2 \bar{\lambda}}{48T^3 \pi^4}, \\ a &= \frac{\zeta(3)}{256\pi^4}(\bar{g}^6 - \frac{\bar{g}^4 \bar{\lambda}}{2} - \frac{\bar{g}^2 \bar{\lambda}^2}{12} + \frac{7\bar{\lambda}^3}{162}), \\ b &= \frac{\zeta(3) \bar{g}^2}{64\pi^4}(-\frac{\bar{g}^4}{4} + \frac{\bar{\lambda} \bar{g}^2}{6} - \frac{\bar{\lambda}^2}{9}), \\ c &= d = 0.\end{aligned}\quad (22)$$

The 1-loop integration of the effective 3D model which takes into account the corrections (19) or (22) will be compared to the solution where the potential is truncated at *dim* 4 level (e.g. eqs.(14) or (20)). We recall, the latter approximation is equivalent to the "daisy-summed" infrared improved treatment of the full 4D thermal theory. The corrections we are going to discuss are formally of higher order than the $\mathcal{O}(\lambda^{3/2})$ and $\mathcal{O}(g^3)$ corrections, which are produced in the 1-loop solution of the effective model, but their actual importance can be assessed only if the physical couplings and $T = 0$ mass values of the W-boson and of the Higgs particle are substituted. It is convenient to compute the 1-loop effective Higgs-potential from the 3D theory in the 3D analogue of the 't Hooft-Landau gauge. We are aware of the fact that especially the higher dimensional pieces of the potential might display some gauge dependence, which, however should not affect physical quantities determined in any fully fixed gauge. Using the notations introduced above the corrected form of the effective potential can be written (almost) uniformly for the U(1) and SU(2) cases:

$$\begin{aligned}U_{3D,eff}^{1-loop} &= \frac{1}{2}(m_\phi^2 + \delta m_\phi^2)\phi_0^2 + \frac{1}{24}(\hat{\lambda} + \delta\lambda)\phi_0^4 + \frac{a}{T^2}\phi_0^6 \\ &- \frac{T}{12\pi}(n_W[Q(g^2 + \delta g^2)]^{3/2}\phi_0^3 + n_{A_0}\bar{m}_D^3 + n_G\bar{m}_G^3 + \bar{m}_H^3),\end{aligned}\quad (23)$$

with $n_W = 6$ (2), $n_{A_0} = 3$ (1), $n_G = 3$ (1) for SU(2) (U(1)), and

$$\begin{aligned}\bar{m}_H^2 &= m_\phi^2 + \delta m_\phi^2 + \frac{1}{2}(\hat{\lambda} + \delta\lambda)\phi_0^2 + 30a\phi_0^4, \\ \bar{m}_G^2 &= m_\phi^2 + \delta m_\phi^2 + \frac{1}{6}(\hat{\lambda} + \delta\lambda)\phi_0^2 + 6a\phi_0^4, \\ \bar{m}_D^2 &= m_D^2 + \delta m_D^2 + Q(g^2 + \delta g^2)\phi_0^2 + 2b\phi_0^4.\end{aligned}\quad (24)$$

($Q = 1/4$ (1) for SU(2) (U(1)).) In eqs. (23) and (24) 4-dimensional notations are reinstated.

The degeneracy temperature of the effective potential (23) has been found in various points of the (λ, g) -plane. Several physical quantities were evaluated for the characterization of the strength

of the phase transition. The order parameter discontinuity was found by locating the non-trivial degenerate minima of the potential at T_c , the relative surface tension σ/T_c^3 was calculated from the formula valid in the thin-wall approximation. Thermal masses at the transition are computed from (24) and

$$m_W^2 = Q(g^2 + \delta g^2)\phi_c^2. \quad (25)$$

In Table 1 results are presented for $G=SU(2)$ with fixed input data ($g = 2/3, v_0 = 241.8$ GeV) and varying $m_H(T = 0)$. These quantities in turn determine $\lambda, m_W(T = 0)$ through the usual relations. All physical quantities are expressed in proportion to the $T = 0$ Higgs condensate v_0 , if not stated otherwise. The results of the first row for each Higgs mass value were obtained from a version of the effective theory with only $\mathcal{O}(T^2)$ mass-corrections and unmodified λ . (This version could be interpreted also as normalised at the common scale $\mu = T$, and assuming the unchanged relationship of the $T=0$ masses and the renormalised couplings granted.) In the second row the results represent the effect induced by the Linde-type renormalisation of the T -independent part of the potential (e.g. eq.(9)). In the third row the ϕ_0^6 -type contributions to the potential are included, finally $\mathcal{O}(m^2/T^2)$ corrections to the masses and λ are taken into account in the entries of the fourth row. All quantities point to the well-known conclusion that the first order nature of the transition weakens with increasing $m_H(T = 0)$.

Numerically important effects come when the logarithmic renormalisation corrections become large. For small (≤ 35 GeV) Higgs masses they make the transition of harder first order nature. They signal large deviation of the physically relevant scale T_c from the normalisation scale v_0 . As a consequence also the effect of the higher dimensional ϕ^6 potential piece becomes important as it is illustrated for $m_H = 20$ GeV in Fig.2. The mass-corrections are everywhere negligible as one recognizes also from the small ϕ region of the figure.

Experimental data seem to exclude the region below 50 GeV for the Higgs mass. Therefore, we conclude that all formally higher order corrections (induced by the renormalisation conditions and/or by taking into account higher dimensional interactions and mass corrections) generated in the reduction procedure are indeed negligible according to the perturbative discussion of the phase transition.

We conclude this section by presenting the evidence for the existence of distinct mass scales within the effective static theory as it emerges from its perturbative solution.

Since this search cannot be based on any *a priori* theoretical guiding principle, it starts with thorough inspection of the thermal mass values of the different fluctuation modes found at T_c perturbatively. Any suggestion for the existence of well-separated scales should be checked for selfconsistency. This means that the solution of a theory resulting from any further reduction, based on the existence of a conjectured mass hierarchy, should preserve that feature in its output.

In Fig. 3 the variation of the thermal masses of the 4 different fluctuations are depicted as functions of $m_H(T = 0)$ at $T = T_c$, as found from (24-25). It is obvious, that m_D is separated from the actual lightest mode(s) everywhere. On the other hand the screening masses of the magnetic gauge and of the scalar modes show an interesting cross-over around 50 GeV. For Higgs masses much below this value it is very plausible to conjecture the existence of a description in terms of a scalar effective theory [11].

With some empathy for $m_H(T = 0) > 80$ GeV the 1-loop perturbative screening mass values can be seen to indicate the relevance of a pure gauge theory, which might faithfully account for the features of the phase transition. In the second part of this study the crudest realisation of the effective pure gauge theory will be presented and its perturbative solution be discussed.

3. Effective Gauge Field – Higgs Field and Pure Gauge Field Theories of the Electroweak Phase Transition

The order of integration over the \bar{A}_0^a and φ -fields will follow the hierarchy of their thermal masses established from Fig.3 at $T = T_c$. Thus first \bar{A}_0^a , then φ , is integrated over. Both integrations can

be performed without fixing a gauge. However, the $\varphi = \varphi_0 = \text{constant}$ and the slowly varying $\bar{A}_i^a(x)$ background fields both break gauge invariance, therefore the result can be interpreted physically only when the gauge is fully fixed. This will be done in section 4.

The integrations will be performed in the Gaussian approximation. Although the use of more sophisticated approaches might prove unavoidable in later investigations, the intuition based on experience in statistical physics tells, that multiple application of relatively inaccurate renormalisation group steps might lead to quite accurate effective dynamics of the lowest modes.

The quadratic form of the part of the action, relevant for the Gaussian \bar{A}_0^a -integration reads, after Fourier-transforming it, as

$$S_{A_0}^{(2)} = \frac{1}{2} \sum_k \bar{A}_0^a [(k^2 + m_D^2 + \frac{1}{4}\bar{g}^2\varphi^2 + \bar{g}^2(A_i^c)^2)\delta^{ab} - 2i\bar{g}\epsilon^{acb}k_i\bar{A}_i^c - \bar{g}^2\bar{A}_i^a\bar{A}_i^b]\bar{A}_0^b. \quad (26)$$

No appealing representation could be found for the determinant of the 3×3 fluctuation matrix, but the first terms of its expansion with respect to \bar{A}_i are as follows:

$$\Delta_{A_0}U[\varphi, \bar{A}_i] = -\frac{1}{4\pi}(m_D^2 + \frac{1}{4}\bar{g}^2\varphi^2)^{3/2} + \frac{\bar{g}^4}{96\pi} \frac{(\bar{A}_i \times \bar{A}_j)^2}{(m_D^2 + \frac{1}{4}\bar{g}^2\varphi^2)^{1/2}} + \mathcal{O}(\bar{A}^6). \quad (27)$$

No mass term is generated in this step for \bar{A} . The quartic piece is interpreted as the correction to the pure gauge kinetic action (no need for explicit derivative expansion!):

$$\begin{aligned} \frac{1}{4}\bar{F}_{ij}^a\bar{F}_{ij}^a &\rightarrow \frac{1}{4\mu}\bar{F}_{ij}^a\bar{F}_{ij}^a, \\ \frac{1}{\mu} &= 1 + \frac{g^2T}{24\pi(m_D^2 + \frac{1}{4}g^2\varphi^2)^{1/2}}. \end{aligned} \quad (28)$$

The effective 3D gauge field-Higgs-model obtained after integrating over \bar{A}_0 is then defined by the following action:

$$\begin{aligned} S[\bar{A}_i, \varphi] &= \int d^3x \left[\frac{1}{4\mu}\bar{F}_{ij}^a\bar{F}_{ij}^a + \frac{1}{2}(D_i\varphi)^+(D_i\varphi) + U(\varphi, \bar{A}_i) \right], \\ U(\bar{A}_i, \varphi) &= \frac{1}{2}m_\phi^2\varphi^+\varphi + \frac{\hat{\lambda}}{24}(\varphi^+\varphi)^2 - \frac{1}{4\pi}(m_D^2 + \frac{1}{4}\bar{g}^2\varphi^2)^{3/2}, \end{aligned} \quad (29)$$

with μ given by eq.(28).

Here we derived the effective action by an explicit integration over the A_0^a field. Another way of proceeding would be to start from the static effective action in eqs. (14)-(17) and to evaluate two Feynman diagrams: the A_0^a loop contribution to the A_i^a and φ propagators. The k^2 term of the former gives the $1/\mu$ in eq. (28) (for $\varphi \ll m_D$) and the latter gives the correction $m_\phi^2 \rightarrow m_\phi^2 - 3\bar{g}^2m_D/(16\pi)$ obtained also from eq. (29) for $\varphi \ll m_D$.

The expansion around φ_0 up to quadratic terms in an \bar{A}_i background gives the following effective masses for the Higgs (H) and the (pseudo)-Goldstone (G) modes:

$$\begin{aligned} \tilde{m}_H^2 &\equiv m_H^2 + \frac{1}{4}\bar{g}^2\bar{A}^2, \quad \tilde{m}_G^2 \equiv m_G^2 + \frac{1}{4}\bar{g}^2\bar{A}^2, \\ m_H^2 &= m_\phi^2 + \frac{\hat{\lambda}T}{2}\varphi_0^2 - \frac{3\bar{g}^2}{16\pi}(m_D^2 + \frac{1}{4}\bar{g}^2\varphi_0^2)^{1/2}, \\ m_G^2 &= m_\phi^2 + \frac{\hat{\lambda}T}{6}\varphi_0^2 - \frac{3\bar{g}^2}{16\pi}[(m_D^2 + \frac{1}{4}\bar{g}^2\varphi_0^2)^{1/2} + \frac{1}{4}\bar{g}^2\varphi_0^2(m_D^2 + \frac{1}{4}\bar{g}^2\varphi_0^2)^{-1/2}]. \end{aligned} \quad (30)$$

It is remarkable that the corrections to the effective masses arising from the expansion of (27) reproduce correctly the A_0 -loop contribution to the scalar selfenergies [3,4].

Integrating finally over φ one obtains the following fluctuation determinant:

$$\Delta_\varphi U(\bar{A}_i) = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \{ \log[(k^2 + \tilde{m}_G^2)(k^2 + \tilde{m}_H^2) - \bar{g}^2(k_i \bar{A}_i)^2] + \log[(k^2 + \tilde{m}_H^2)^2 - \bar{g}^2(k_i \bar{A}_i)^2] \}, \quad (31)$$

which after the expansion into powers of \bar{A}_i yields the following contribution to the pure gauge action:

$$\begin{aligned} \Delta_\varphi U(\bar{A}_i) = & -\frac{1}{12\pi}(m_H^3 + 3m_G^3) + \frac{1}{2}\bar{A}^2 \frac{\bar{g}^2}{48\pi} \frac{(m_G - m_H)^2}{m_G + m_H} \\ & + \frac{\bar{g}^4}{48\pi} (\bar{A}_i \times \bar{A}_j)^2 \left[\frac{1}{5(m_G + m_H)} + \frac{m_G m_H}{5(m_G + m_H)^3} + \frac{1}{8m_G} \right] \\ & - \frac{\bar{g}^4 \bar{A}^4 (m_G - m_H)^2}{2560\pi m_G m_H (m_G + m_H)^3} [5(m_G^2 + m_H^2) + 6m_G m_H] + \mathcal{O}(\bar{A}^6). \end{aligned} \quad (32)$$

The first term on the right hand side of (32) is the usual 1-loop contribution of the scalar fluctuations to the effective ϕ_0 -potential within the 3D theory. The second and the fourth terms (and probably many of the higher order terms too) violate gauge invariance. These terms are proportional to $(m_G - m_H)^2$, what is clear manifestation of the fact that they are induced by the symmetry breaking value of the ϕ -field. It is remarkable again that the second term reproduces the contribution of the scalar loops to the polarisation tensor of the magnetic gauge fluctuations [3,4].

It seems worthwhile to comment on the stability of the ground state of eq. (32) with respect to A^2 -fluctuations. In a fluctuating A -background it is more appropriate to reexpress (32) in terms of the field dependent masses \tilde{m}_G, \tilde{m}_H , defined in (30). This replacement to $\mathcal{O}(A^4)$ affects directly only the coefficient of A^2 in (32). Then one recognizes that the $\sim A^4$ terms absorbed in this way from the higher power terms turn the sign of the $\mathcal{O}(A^4)$ term in the last line of (32) into positive. Therefore (32) actually does not signal any vacuum instability in the direction of the vector potential fluctuations. The same argument holds also for the U(1) case in view of the remarks at the end of this section.

From the third term of (32) one finds the contribution of the scalar fields to the magnetic susceptibility of the gauge medium. This term is proportional to T/m_G , therefore it requires careful monitoring, in order to avoid the breakdown of the proposed approximation, at least not for this specific reason.

For the discussion of the next section it is somewhat more convenient to work with the usual form of the gauge kinetic term. This is achieved by a finite rescaling of the vector potential, compensated in the potential terms by a screened gauge coupling:

$$\tilde{g}^2 = \frac{g^2}{1 + \frac{g^2 T}{48\pi} \left(\frac{1}{2m_G} + \frac{4}{5(m_G + m_H)} + \frac{4m_G m_H}{5(m_G + m_H)^3} + \frac{2}{(m_D^2 + \frac{1}{4}g^2 \phi_0^2)^{1/2}} \right)}. \quad (33)$$

In terms of the rescaled quantities the action of the effective pure gauge action has the following form:

$$\begin{aligned} S[\bar{A}_i] = & \int d^3 x \left\{ \frac{1}{4} \bar{F}_{ij}^a \bar{F}_{ij}^a + \frac{\tilde{g}^2 T}{2} \bar{A}_i^2 \left(\frac{1}{4} \varphi_0^2 + \frac{(m_G - m_H)^2}{48\pi(m_G + m_H)} \right) \right. \\ & \left. - \frac{\tilde{g}^4 T^2 (\bar{A}^2)^2 (m_G - m_H)^2}{2560\pi m_G m_H (m_G + m_H)^3} [5(m_G^2 + m_H^2) + 6m_G m_H] + \mathcal{O}(\bar{A}^6) + \text{gauge fixing terms} \right\}. \end{aligned} \quad (34)$$

The effective theory in the broken phase differs from the 3D QCD essentially because of the presence of φ_0 -induced terms. It is an interesting question to devise appropriate non-perturbative methods for the study of the gauge dynamics in this phase.

Finally for completeness we quote the formulae resulting from a similar procedure applied to the Abelian Higgs model. Since the additional Higgs fields for the non-Abelian case actually contribute only to the magnetic susceptibility (cf. the contribution of the second term of (31)), the effective pure gauge action is of the same form after a replacement $g/2 \rightarrow g$ is made and the appropriate screened coupling and thermal masses are used in it:

$$\tilde{g}^2 = \frac{g^2}{1 + \frac{g^2 T}{15\pi} \left(\frac{1}{m_G + m_H} + \frac{m_G m_H}{(m_G + m_H)^3} \right)}, \quad (35)$$

$$\begin{aligned} m_H^2 &= m_\phi^2 + \frac{\hat{\lambda}}{2} \phi_0^2 - \frac{g^2 T}{4\pi} [(g^2 \phi_0^2 + m_D^2)^{1/2} + g^2 \phi_0^2 (g^2 \phi_0^2 + m_D^2)^{-1/2}], \\ m_G^2 &= m_\phi^2 + \frac{\hat{\lambda}}{6} \phi_0^2 - \frac{g^2 T}{4\pi} (g^2 \phi_0^2 + m_D^2)^{1/2}, \end{aligned} \quad (36)$$

(the formulae for m_D^2, m_ϕ^2 are given in (21)).

The essential characteristics of the perturbative solution of the SU(2) and U(1) cases are expected very similar, therefore in the short section 4 we restrict our study to the non-Abelian case.

4. Perturbative Discussion of the Effective Pure Gauge Theory

In this section the 1-loop integration of the effective pure gauge action is discussed in the Landau gauge. It is worthless to say, that conclusions drawn from this approximate solution are of very restricted conceptual interest.

In the Landau gauge the six transverse degrees of gauge freedom fluctuate with the thermal mass

$$m_W^2 = \frac{1}{4} \tilde{g}^2 \phi_0^2 + \frac{\tilde{g}^2 T}{48\pi} \frac{(m_H - m_G)^2}{m_H + m_G}. \quad (37)$$

The fluctuations of the spatial longitudinal and the ghost degrees of freedom are infinitely suppressed. At 1-loop level the higher power \bar{A} -terms do not play any role.

The effective Higgs potential is obtained by adding to the gauge contribution the \bar{A} -independent pieces from eqs. (27) and (32), and also from the potential part of (14). The result is

$$U_{eff}(\phi_0) = \frac{1}{2} m_\phi^2 \phi_0^2 + \frac{\hat{\lambda}}{24} \phi_0^4 - \frac{T}{12\pi} [6m_W^3 + 3(m_D^2 + \frac{1}{4} g^2 \phi_0^2)^{3/2} + 3m_G^3 + m_H^3]. \quad (38)$$

On the basis of Fig.3 the proposed effective description is expected to work for higher m_H values. Therefore in Table 2 we compare physical quantities characterising the phase transition for $m_H = 80, 100, 120$ GeV as given by the potential (38) with the results from the conventional perturbative treatment of the full 3D effective theory. The parameters $m_W(T=0) = 80.6$ GeV, $g = 2/3$ are kept fixed close to their experimental values.

First of all we can state that by the observed values of the screened coupling the approximation is not endangered in this $m_H(T=0)$ region by T/m_G growing too large. The general characteristics of all the physical quantities hints to stronger discontinuities from the effective gauge theory relative to the usual perturbative treatment. The decrease in T_c is 2-3%, the increase of ϕ_c/v_0 is 5-10%, and that of σ/T_c^3 is $\approx 50\%$.

On the other hand the gap between the scalar and the vector screening masses closes considerably, at least in the present *perturbative* solution of the effective pure gauge theory (Fig. 4). This fact can be traced mainly to the negative mass contribution of the \bar{A}_0 -loop to the scalar masses (eq.(30)). Similar phenomenon led the authors of Ref.[3] to restrict the validity of the improved 1-loop perturbative treatment to $m_H(T=0) < 100$ GeV. It could be that the opening of the gap is just shifted to larger Higgs masses.

Finally, we should compare our findings with the available results of non-perturbative investigations of the reduced 3D model [8,9]. For this purpose their data obtained at $m_H = 80 \text{ GeV}$ seem to be the most relevant: $T_c = 160 \text{ GeV}$, $\phi_c/T_c = 0.67$. Both differ considerably from the perturbative results, independently of whatever version of that latter is being considered. In the effective pure gauge model (with the renormalisation conditions (9)) one has $T_c = 184.5 \text{ GeV}$, $\phi_c/T_c = 0.3$, which still is shifted slightly towards the MC results from the output of the usual perturbative treatment.

This circumstance underlines once again the interest of a non-perturbative study of the effective pure gauge model. Within the perturbative approach, higher loop corrections were claimed to provide essential modification of the phase transition characteristics [14]. Two-loop calculations relevant to the analysis of the 3D effective models are in progress [15,16].

5. Conclusions

The interest in the three dimensional reduced effective theory of the finite temperature electroweak interactions is obvious even from the great simplification occurring in its non-perturbative study versus the simulation of the full four-dimensional formulation [8,9]. Also attempts to exploit the extensive experience with critical phenomena in 3 dimensions need a most faithful mapping onto a formally $T = 0$ effective 3D theory [17].

In this paper we have studied a hierarchy of effective field theories for hot electroweak matter. The hierarchy is based on integrating subsequently over fields following the hierarchy of their masses.

The first stage was the standard dimensional reduction [6] in which nonstatic bosonic modes with temporal Fourier index $n \neq 0$ and with an effective mass of $2\pi nT$ were integrated over. This leads to the effective action $S[A_i^a, A_0^a, \varphi]$ given in eqs. (14)-(17). We carried out the integration in a new way, which permitted us to relate the 4D and 3D couplings and to study the magnitudes of higher dimensional operators multiplied by inverse powers of T (for dimensional reasons). We have restricted the range of importance of the $\mathcal{O}(\log(m^2/T^2))$ and $\mathcal{O}(m^2/T^2)$ terms of the effective action to well below the relevant experimental range of the mass of the Higgs particle ($m_H \leq 35 \text{ GeV}$). The line of thought presented here can be followed without any change also for the more complete version of the model, including the $U(1)$ -component of the gauge field.

Subsequent stages were based on the hierarchy of masses exhibited in Fig. 3. First, due to the large Debye mass m_D , the A_0 field is integrated over, leading to the $S[A_i, \phi]$ given in eq.(29). The order of further integrations is not so obvious. For Higgs masses above the present lower limit it seems more accurate to integrate over the Higgs field, leading to the effective action $S[A_i]$ given in eq. (34). For smaller Higgs masses integration over A_i leads to an $S[\phi]$ studied in [11]. Tendency for the hardening of the gauge-field driven phase transition could have been observed when the final effective pure gauge theory has been solved perturbatively for $m_H(T = 0) \geq 80 \text{ GeV}$.

These rather reassuring results urge for the continuation of our investigation in different directions. First of all, it would be very important to have a systematic study of the gauge dependence of the reduced action, and be able at the same time to argue for the gauge independence of the phase transition characteristics on the basis of explicit calculations. The second question of interest refers to the importance of higher loop effects both in the reduction and in the solution of the effective theory. Thirdly, at least the integration over the screened static A_0 field should be possible to perform with an accuracy equal or better than that of the non-static modes and its indirect effects on the effective Gauge-Higgs dynamics be studied. Possible application of some sort of low- k cut-off integration technique [18] might prove useful in this respect.

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Appendix

For the simple case of $A_0 = 0$ the matrices K_i of eq.(6) take block-diagonal form further facilitating the evaluation of their determinants:

$$\det K_1 = \left(\det \begin{pmatrix} \omega_n^2 + M_W^2 & \frac{ig}{2} k \Phi \\ \frac{-ig}{2} k \Phi & K^2 + M_G^2 \end{pmatrix} \right)^2, \quad (A.1)$$

$$\det K_2 = (K^2 + M_H^2) \det \begin{pmatrix} \omega_n^2 + M_W^2 & \frac{ig}{2} k \Phi \\ \frac{-ig}{2} k \Phi & K^2 + M_G^2 \end{pmatrix}, \quad (A.2)$$

$$\det K_3 = (K^2 + M_W^2)^2, \quad \det K_4 = K^2 + M_W^2 \quad (A.3)$$

$$(M_W^2 = g^2 \Phi^2/4, \quad M_H^2 = m^2 + \lambda \Phi^2/2, \quad M_G^2 = m^2 + \lambda \Phi^2/6).$$

The 1-loop correction to the effective potential in the thermal static gauge is written as

$$\Delta U = \int_k' \left\{ \frac{3}{2} \ln((K^2 + M_G^2)(\omega_n^2 + M_W^2) - k^2 M_W^2) + \frac{1}{2} \ln(K^2 + M_H^2) + 3 \ln(K^2 + M_W^2) \right\}, \quad (A.4)$$

where the meaning of the symbol $\int_k' = T \sum_{n \neq 0} \int d^3 k / (2\pi)^3$.

One is allowed to expand the logarithms in powers of M_i^2 . We work up to $\mathcal{O}(M_i^4)$ allowing to find the 1-loop corrections to the classical Higgs potential. The field independent leading terms can be omitted and the "primed" integrals of (A.4) are performed with help of the following relations:

$$\begin{aligned} \int_k' \frac{1}{K^2} &= \frac{\Lambda^2}{8\pi^2} - \frac{\Lambda T}{2\pi^2} + \frac{T^2}{12}, \\ \int_k' \frac{1}{K^4} &= \frac{1}{8\pi^2} + \frac{D_0}{8\pi^2}, \\ \int_k' \frac{1}{\omega_n^2 K^2} &= \frac{\Lambda}{24\pi^2 T} - \frac{D_0}{4\pi^2}, \end{aligned} \quad (A.5)$$

with

$$D_0 = \sum_{n \neq 0} \int_0^{\Lambda\beta} dx \frac{2}{x^2 + (2\pi n)^2} \sim \ln \frac{\Lambda}{T}. \quad (A.6)$$

These steps lead to the detailed formula

$$\begin{aligned} \Delta U &= \left(\frac{9}{2} M_W^2 + \frac{3}{2} M_G^2 + \frac{1}{2} M_H^2 \right) \left(\frac{\Lambda^2}{8\pi^2} - \frac{\Lambda T}{2\pi^2} + \frac{T^2}{12} \right) \\ &+ \frac{1 - D_0}{16\pi^2} \left[\frac{3}{2} (M_W^2 + M_G^2)^2 + \frac{1}{2} M_H^4 + 3 M_W^4 \right] - \frac{3 D_0}{8\pi^2} M_W^2 M_G^2 \\ &= \Phi_0^2 \left[\left(\frac{9g^2}{64\pi^2} + \frac{\lambda}{16\pi^2} \right) \Lambda^2 - \left(\frac{9g^2}{8} + \frac{\lambda}{2} \right) \frac{\Lambda T}{2\pi^2} + \left(\frac{9g^2}{8} + \frac{\lambda}{2} \right) \frac{T^2}{12} + m^2 \left(\frac{\lambda}{16\pi^2} + \frac{3g^2}{64\pi^2} \right) \right. \\ &\quad \left. - D_0 m^2 \left(\frac{\lambda}{16\pi^2} + \frac{9g^2}{64\pi^2} \right) \right] \\ &+ \Phi_0^4 \left[\frac{\lambda^2}{96\pi^2} + \frac{9g^4}{512\pi^2} + \frac{\lambda g^2}{128\pi^2} - D_0 \left(\frac{\lambda^2}{96\pi^2} + \frac{9g^4}{512\pi^2} + \frac{3\lambda g^2}{128\pi^2} \right) \right] + \Phi \text{ indep. terms} \end{aligned} \quad (A.7)$$

The logarithmic divergences originate from terms proportional to D_0 .

Table Captions

Table 1 Phase transition characteristics from various truncations of the effective 3D theory a – *dim 4* terms with $\mu = T$ renormalisation scale, b – *dim 4* terms with Linde-type renormalisation conditions, c – *dim 6* terms with Linde type renormalisation conditions, d – *dim 6* terms plus $\mathcal{O}(m^2/T^2)$ mass and λ corrections with Linde-type renormalisation conditions

Table 2 Phase transition characteristics from the 3D pure gauge theory with $\mu = T$ renormalisation scale (b), and with Linde-type renormalisation condition (c) compared with case (a) of the previous table

Figure Captions

- Fig. 1 Contour lines of the relative variation of a) $(\hat{m}^2 - m^2)/m^2$, b) $(\hat{\lambda} - \lambda)/\lambda$ covering the $m_H - T$ range relevant to the phase transition. The various lines correspond to the 3%, 5%, 7% levels, respectively.
- Fig. 2 Comparison of the effective potentials for the U(1) Higgs model with *dim 4* and *dim 6* truncation for $m_H(T = 0) = 20\text{GeV}$ at the transition temperature of the *dim 6* truncated model
- Fig. 3 Thermal (screening) masses of different fluctuations at $T = T_c$ as functions of m_H in the SU(2) Higgs model ($v_0 = 241.8\text{GeV}$, $g = 2/3$) calculated perturbatively from the *dim 4* truncated effective theory (Linde-type renormalisation conditions)
- Fig. 4 Thermal (screening) masses of different fluctuations at $T = T_c$ as functions of m_H in the SU(2) Higgs model ($v_0 = 241.8\text{GeV}$, $g = 2/3$) as calculated perturbatively from the effective pure gauge theory.

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